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Proper Lucky Labeling of Butterfly, Benes and Hypercube Networks

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Abstract

In this paper we investigate the proper lucky labeling of butterfly network, benes network, and cube connected cycle.

Keywords: Butterfly network, benes network, cube centred cycle and proper lucky labeling.

1. Introduction:

Graph labeling is the mapping of graph elements to the set of integers. Rosa introduced graph labeling in 1967. A. Ahai et al and S. Akbari et al studied the lucky labeling of graphs. Proper lucky labeling of graphs is introduced by Thivyarati et al and kujur et al. They studied proper lucky number of bloom graph, hexagonal mesh and honey comb network , k identified triangular mesh , k identified sierpinski network and certain tree families.

In this paper, we determine the proper lucky number of butterfly network, benes network and cube centred cycle.

2. Preliminaries:

Let $G(V,E)$ be a graph, where V be the vertex set and E be the edge set.

Definition1: The neighbourhood of a vetex v is the set of all vertices which are adjacent to v .

Definition 2: Let f be a mapping of vertices of G to the set of natural numbers. Define the sum of neighbourhood of vertex v by $s(v) = \sum_{v\in N(v)} f(v)$, where $N(v)$ denotes the open neighbourhood of vertex of $v \in V$. A labeling f is a lucky labeling if $s(u) \neq s(v)$ for all $uv \in E(G)$.

Definition 3: A labeling f is a proper lucky labeling if $f(u) \neq f(v)$ and $s(u) \neq s(v)$ for all $uv \in E(G)$. The proper lucky number of G, denoted by $\eta_p(G)$ is the least positive integer k such that G has a proper lucky labeling with $\{1,2, \dots ,k\}$ as the set of labels.

Main results:

Theorem 1: Let $BF(n)$ be a n dimensional butterfly network, then its proper lucky number $\eta_p(BF(n)) = \begin{cases} 2 \text{ if } n \text{ is even} \\ 3 \text{ if } n \text{ is odd} \end{cases}$

Proof:

The n dimensional *butterfly network*, denoted by $BF(n)$, has a vertex set $V = \{(x,i); x \in V(Q_n), 0 \leq i \leq n\}$ .Two vertices (x,i) and (y,i) are linked by an edge in $BF(n)$ if and only if $j = i + 1$ and either (i). $x = y$ or

(ii). *x* differs from *y* in precisely the j^{th} bit.

For $x = y$, the edge is said to be a straight edge. Otherwise the edge is a cross edge. For fixed *i* the vertex $(x; i)$ is a vertex on level *i*.

The butterfly network has $(0, 1, 2, \dots, n)$ levels and each level has 2^n vertices. Label the vertices in level of i as 2 and level $i + 1$ as 1, $0 \leq i \leq n$ so on In particular odd level vertices are labelled as 1 and even level vertices are labelled as 2. The degree of the vertices in level 0 and level n is 2 and in other levels it is 4. Each vertex in level i $1 \leq i \leq n-1$, has its neighbours in level $i-1$ and level $i+1$. In other way

every vertex in odd level has its neighbours in even level and vice versa

Case 1: When n is even

Sub case 1: Suppose u is a vertex in an odd level i , then its neighbours v are in even level and $f(v) = 2$,, then for every vertex u ,$s(u) = 2 + 2 + 2 + 2 = 8$ and v has its neighbours in odd level then $s(v) = 1 + 1 + 1 + 1 = 4$. Here $f(u) \neq f(v)$ and $s(u) \neq s(v)$

Sub case 2: Suppose u is a vertex in an even level, then its neighbours v are in odd level and $f(v) = 1$, then for every vertex u , $s(u) = 1 + 1 + 1 + 1 = 4$.The neighbours u of v are in even level and $f\ (u) = 2$, $s(v) = 2 + 2 + 2 + 2 = 8$, and $f(u) \neq f(v)$ and $s(u) \neq s(v)$.

Suppose u is a vertex in n^{th} level then its neighbours v are at odd level and $f(v) = 1$, then for every vertex u , $s(u) = 1 + 1 = 2$. The vertex v has its neighbours u in even level, $f(u) = 2$ and for every vertex v , $s(u) = 2 + 2 = 4$ .Hence $f(u) \neq f(v)$ and $s(u) \neq s(v)$.The same argument is true for the vertices in 0 level.

In every level for an edge uv, $f(u) \neq f(v)$ and $s(u) \neq s(v)$. Hence the labeling is proper lucky labeling and the proper lucky number of butterfly network is 2.

Case 2: When n is odd

Label the vertices of the butterfly network as in the previous case. All odd level vertices are labeled as 1 and even level vertices are labeled as 2.The vertices in level 0 are labeled as 2.Suppose u is in level n then its neighbours v are in even level and $f(v) = 2$. Then for every vertex u ,$s(u) = 2 + 2 = 4$. The vertex v has its neighbours u in odd level and $f(u) = 1$. Then for every vertex v in level $n - 1$,$s(v) = 1 = 1 + 1 + 1 = 4$. Here $f(u) \neq f(v)$, then the labeling is not proper lucky labeling. Therefore we need more than 2 labels when n is odd, and Label the vertex v in level $n - 1$ as $f(v) = \begin{cases} 2, 1 \leq j \leq 2^{n-1} \\ 3, \quad 2^{n-1} \leq j \leq 2^n \end{cases}$. For every vertex u in level n , $s(u) = 2 + 3 = 5$ and for every vertex in level $n - 1$, $s(v) = 1 + 1 + 1 + 1 = 4$, $f(u) \neq f(v)$ and $s(u) \neq s(v)$. Hence the labelling is the proper lucky labelling and the proper lucky number of butterfly network is 3.

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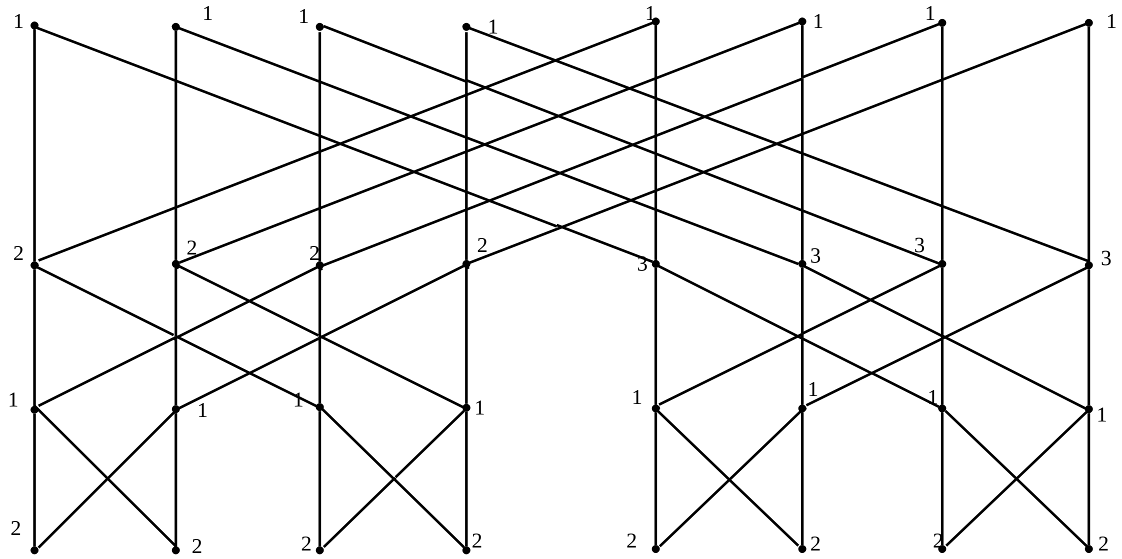


$Fig\ 1: Butterfly\ network\ BF(3)$

Theorem 2: Proper lucky number of $BB(n) = 2$

Proof: The n dimensional *benes network BB* (*n*) consists of back to back butterfly network and it has $2n + 1$ levels, $(2n + 1)2^n$ vertices and $n2^{n+2}$ edges. The benes network has $2n + 1$ levels including level 0.Benes network is obtained by merging two butterfly networks at the zero level. Label the vertices in odd level as 1 and even level as 2. The vertices in each level i has its neighbours in level $i - 1$ and level $i + 1$,$1 \leq i \leq 2n$. Vertex in level 0 has its neighbours in level 1 and vertex in level n has its neighbours in level $n - 1$. Degree of vertex in level 0 and level 2n is 2 and degree of the rest of the vertices is 4 .

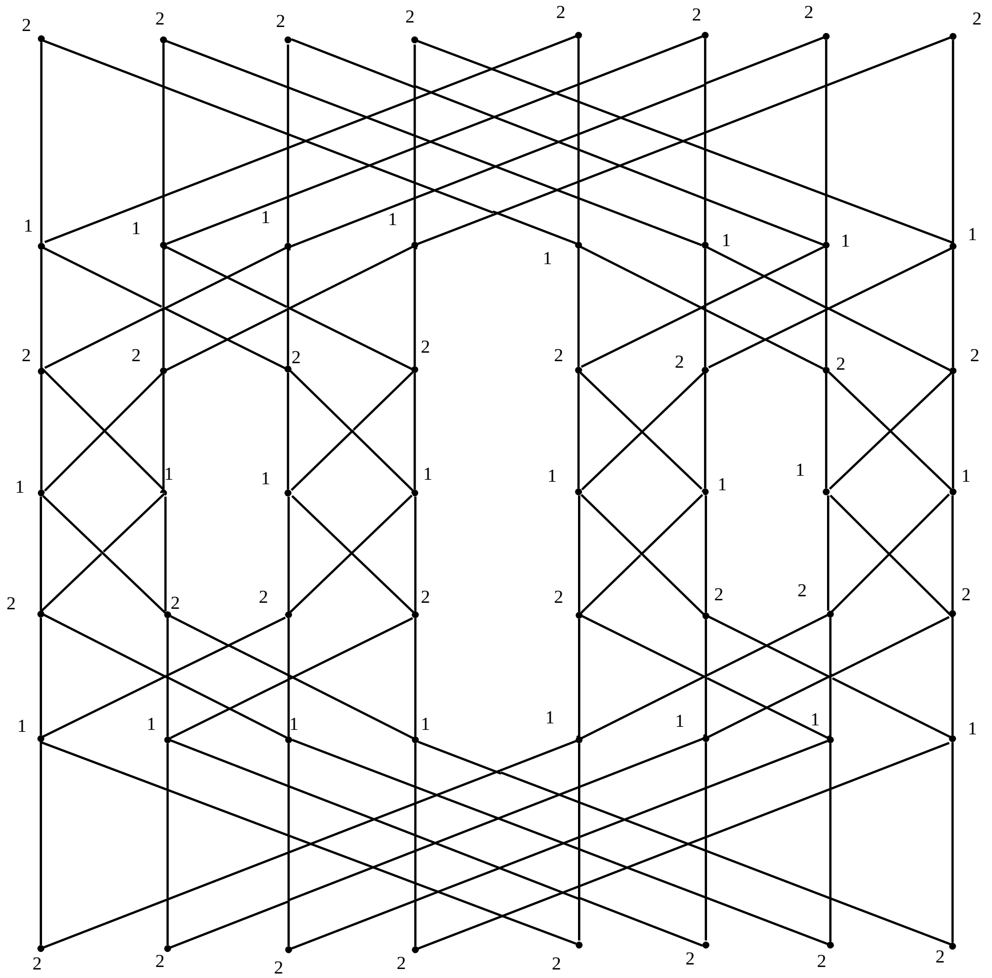
Case 1: Suppose u is a vertex in level 0, which has two of its neighbours in level1, $v \in N(u), f(v) = 1$. For every vertex in level 0, $s(u) = 1 + 1 = 2$. The vertex v in level 1 has its neighbours in level 0 and level 2. $f(u) = 2, s(v) = 2 + 2 + 2 + 2 = 8$. So $f(u) \neq f(v)$ and $s(u) \neq s(v)$.

Case 1: Suppose u is a vertex in an odd level i , then its neighbours v are in even level and $f(v) = 2$, then

for every vertex u ,$s(u) = 2 + 2 + 2 + 2 = 8$ and v has its neighbours in odd level then $s(v) = 1 + 1 + 1 + 1 = 4$. Here $f(u) \neq f(v)$ and $s(u) \neq s(v)$..

Case 2: Suppose u is a vertex in an even level, then its neighbours v are in odd level and $f(v) = 1$, then for every vertex u , $s(u) = 1 + 1 + 1 + 1 = 4$.The neighbours u of v are in even level and $f(u) = 2$, $s(v) = 2 + 2 + 2 + 2 = 8$, and $f(u) \neq f(v)$ and $s(u) \neq s(v)$. Suppose u is a vertex in level $2n$ then its neighbours v are at odd level and $f(v) = 1$, then for every vertex u , $s(u) = 1 + 1 = 2$. The vertex v has its neighbours u in even level, $f(u) = 2$ and for every vertex v , $s(u) = 2 + 2 = 4$ .Hence $f(u) \neq f(v)$ and $s(u) \neq s(v)$.The same argument is applicable for 0 level vertices also.

In every level, for an edge uv, $f(u) \neq f(v)$ and $s(u) \neq s(v)$. Hence the labelling is proper lucky labelling and the proper lucky number of benes network is 2.



$Fig\ 2: BB(3)$

Theorem 4: Proper lucky number Q_n is 2.

Proof: The *n dimensional hyper cube* denoted by $Q(d_1, d_2, \dots, d_n)$ and the vertex set

$V = \{x_1, x_2, \dots, x_n : x \in \{0, 1, 2, \dots . d_i - 1\}, i = 1, 2, 3 \dots n\}$ and two vertices

$x = x_1 x_2 \dots x_n$ and $y = y_1 y_2 \dots y_n$ are linked by an edge if and only if they differ exactly in one coordinate.

Label the vertices of each face of the hypercube Q_n with 1 and 2 so that diagonally opposite vertices receive the same label. Suppose $f(u) = 1$ and $\forall\, v \in N(u), f(v) = 2$. Then $s(u) = 2 + 2 + 2 \dots + 2\ (n\ times) = 2n$ and $\forall\, u \in N(v), f(v) = 1$. Then $s(u) = 1 + 1 \dots + 1(\text{n times})$. So for every edge $uv \in Q_n$,$f(u) \neq f(v)$ and $s(u) \neq s(v)$. Hence the labeling is a proper labeling and the proper lucky number Q_n is 2

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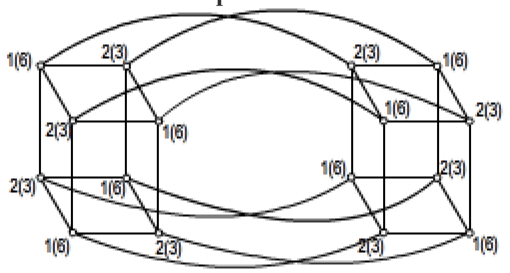


$Fig\ 4: Hypercube\ Q_4$

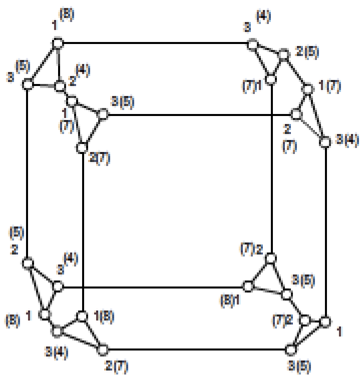
Theorem 5: Proper lucky number of $CCC(3)$ is 3.

Proof: The *n dimensional cube connected cycle*, denoted by $CCC(n)$, is constructed from the n- dimensional hypercube Q_n by replacing each vertex of Q_n with undirected cycle of length n. The vertex set of $CCC(n)$ is V= $\{(x;i); x \in V(Q_n), 1 \leq i \leq n\}$. Two vertices $(x;i)$ and $(y;j)$ are linked by an edge in $CCC(n)$ if and only if either

(i) $x = y$ and $|i - j| \equiv 1(mod\ n)$ or

(ii) $i = j$ and x differs from y in precisely the i^{th} bit.

Edges of the first type are called cycle edges while edges of the second type are called as hypercube edges.Label the vertices of cube connected cycle as 1, 2 and 3 as in the figure.6 Every adjacent vertices of it has distinct labels and different sum. Hence cube connected cycle admits proper labeling and its proper lucky number is 3.



$Fig\ 6: Cube\ connected\ cycle\ CCC(3)$

Conclusion: Proper lucky labeling of butterfly network, benes network, pyramid network, hyper cube network, cube connected cycle and hexagonal mesh pyramid network were discussed and their proper lucky number was obtained.

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