



Effect of Heat Transfer on MHD flow of Casson fluid in an unsteady Parallel Porous Channel with Radiation and Chemical Reaction

D. Vidyanadha Babu

Dept. of mathematics,
QIS College of Engineering & Technology,
Ongole, Prakasham(dt), 523272, A.P. India

Abstract

The influence of heat transfer on MHD flow of Casson fluid in an unsteady porous channel in the presence of heat source and chemical reaction is investigated. The Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Similarity transformations are employed to transform the governing partial differential equations into ordinary differential equations. Then the dimensionless governing equations are solved analytically by using regular perturbation method. The effects of various parameters namely Grasshof number, modified Grasshof number, Radiation parameter, Chemical reaction parameter, Schmidt number, Peclet number, rate of heat transfer Sherwood number, Hartman number on velocity, temperature and concentration fields are presented graphically and discussed.

Keywords: MHD, Casson fluid, chemical reaction, Porous channel, thermal radiation.

Introduction : Non Newtonian fluid flows play a vital role in various branches of science, engineering, and technology. These are used in chemical and nuclear industries, material processing, geophysics, and bio-engineering. has gained much more attention of engineers and scientist due to its important application. Casson fluid is also a non Newtonian fluid . It exhibits the properties of non-Newtonian fluids. Due to the complexity of these fluids, there is no single constitutive equation which exhibits all properties of such non-Newtonian fluids. Several investigators [1-6] has been developed different models to find the MHD behavior of non Newtonian fluids . Many researchers effectively works on casson fluid flows with heat transfer through porous medium [6-20]. Chamkha ,Aly and Mansour [21] analyzed Similarity solutions for unsteady heat and mass transfer from a stretching surface embeded in a porous medium with suction or injection and chemical reaction effects. Rajagopal [22] studied the Stoke's problem for a non Newtonian fluid. KD.Singh [23] find an exact solution of an oscillatory MHD flow in a channel filled with porous medium. Chamkha, Aly and Mansour extracted Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. [30]

2. Mathematical formulation

Consider the unsteady two-dimensional flow and heat transfer of an incompressible Casson fluid over an exponentially shrinking/stretching porous sheet in the presence of radiation and heat source with chemical reaction at $y=0$ with flow being confined in $y > 0$. The fluid is electrically



conducting in the presence of uniform magnetic field applied normal to the sheet, and the induced magnetic field is neglected under the approximation of small Reynolds number.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is [24, 25]

$$\tau_{ij} = \begin{cases} \left(\mu_B + \frac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi > \pi_c \\ \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) 2e_{ij}, & \pi < \pi_c \end{cases} \quad (2.1)$$

where μ_B is plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of fluid, π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the $(i, j)^{\text{th}}$ component of the deformation rate, and π_c is critical value of π based on non-Newtonian model.

If u and v are the velocity components in x and y directions respectively at time t in the flow field, then the two dimensional boundary layer equations in the presence of transverse magnetic field in the presence of chemical reaction as

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \sigma \frac{B_0^2 u}{\rho} + g\beta_T(T - T_0) + g\beta_c(C - C_0) \quad (2.2)$$

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + Q \frac{(T - T_0)}{\rho c_p} \quad (2.3)$$

$$\frac{\partial C}{\partial t} = D_a \frac{\partial^2 C}{\partial y^2} - K_r (C - C_0) \quad (2.4)$$

The boundary conditions are for velocity, temperature and species concentration field as follows.

$$u = 0, \quad T = T_0 \text{ and } C = C_0 \text{ on } y = 0$$

$$u = 0, \quad T = T_w \text{ and } C = C_w \text{ on } y = 1 \quad (2.5)$$

Where ν is kinematic fluid viscosity, ρ is the fluid density, μ is viscosity of the fluid, $\gamma = \mu_B \sqrt{2\pi c} / p_y$ is the casson parameter., σ is electrical conductivity of the fluid and B_0 is the strength of magnetic field applied in the y direction. c_p is the specific heat, K_T is thermal conductivity. Q is the quantity of the heat, T is the temperature, and β_T coefficient of thermal expansion, β_c coefficient of mass expansion, K_r is the concentration parameter, D_a Darcy number, K is porous medium permeability coefficient.

Assume that the fluid is optically thin with a relatively low density and radiative heat flux is according to the Ref [38] is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_0 - T) \quad (2.6)$$



Now introducing non dimensional variables as follows.

$$x^* = \frac{x}{a}, \quad y^* = \frac{y}{a}, \quad u^* = \frac{u}{U}, \quad Re = \frac{Ua}{\nu}, \quad \theta = \frac{T - T_0}{T_w - T_0},$$

$$H^2 = \frac{a^2 \sigma_e B_0^2}{\rho \nu}, \quad t^* = \frac{tU}{a}, \quad P^* = \frac{aP}{\rho \nu U}, \quad Da = \frac{K}{a^2}, \quad Gr = \frac{g \beta_T (T_w - T_0) a}{U \nu},$$

$$G_c = \frac{g \beta_c (C_w - C_0) a^2}{U \nu}, \quad Pe = \frac{U a \rho C_p}{k}, \quad C = \frac{C - C_0}{C_w - C_0}, \quad N^2 = \frac{4a^2 \alpha^2}{k}, \quad S^2 = \frac{1}{Da},$$

$$E = \frac{Q a^2}{k}, \quad J = \frac{K_r a}{U} \quad (2.7)$$

After neglecting the * symbols, the dimensionless governing equations together with the appropriate boundary conditions can be written as

$$Re \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - (S^2 + H^2)u + Gr \theta + G_c C \quad (2.8)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + E\theta \quad (2.9)$$

$$\frac{\partial C}{\partial t} = Sc \frac{\partial^2 C}{\partial y^2} - JC \quad (2.10)$$

The dimensionless boundary conditions

$$u = 0, \quad \theta = 0 \text{ and } C = 0 \text{ on } y = 0$$

$$u = 0, \quad \theta = 1 \text{ and } C = 1 \text{ on } y = 1 \quad (2.11)$$

III METHOD OF SOLUTION

To solve equations (2.8), (2.9) and (2.10) assume

$$\frac{\partial p}{\partial x} = \lambda e^{i\omega t}, \quad u(y, t) = u_0(y) e^{i\omega t}, \quad \theta(y, t) = \theta_0(y) e^{i\omega t}, \quad C(y, t) = C_0(y) e^{i\omega t} \quad (3.1)$$

Substituting equation (2.12) in equations (2.8), (2.9) and (2.10)

$$\left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u_0}{\partial y^2} - m_2^2 u_0 = -\lambda - Gr \theta_0 - G_c C_0 \quad (3.2)$$

$$\frac{d^2 \theta_0}{dy^2} + m_1^2 \theta_0 = 0 \quad (3.3)$$

$$\frac{d^2 C_0}{dy^2} - m_3^2 C_0 = 0 \quad (3.4)$$



Subject to the boundary conditions

$$u_0 = 0, \quad \theta_0 = 0 \text{ and } C_0 = 0 \text{ on } y = 0$$

$$u_0 = 0, \quad \theta_0 = 1 \text{ and } C_0 = 1 \text{ on } y = 1 \quad (3.5)$$

The analyzed solution of equations (3.2) – (3.5) with satisfy boundary conditions (3.5) are given by

$$u(y, t) = \left(\frac{\gamma}{1 + \gamma} \right) \left\{ \begin{aligned} & \frac{G_r}{m_1^2 - m_2^2} \left(\frac{\sinh m_2 y}{\sinh m_2} - \frac{\sin m_1 y}{\sin m_1} \right) - \frac{G_c}{m_3^2 - m_2^2} \left(\frac{\sinh m_2 y}{\sinh m_2} - \frac{\sinh m_3 y}{\sinh m_3} \right) + \\ & \frac{\lambda}{m_2^2} \left(\frac{\sinh m_2 y}{\sinh m_2} (1 + e^{m_2}) \right) + \frac{\lambda}{m_2^2} ((1 + e^{m_2})) \end{aligned} \right\} e^{i\omega t},$$

$$\theta(y, t) = \left(\frac{\sin m_1 y}{\sin m_1} \right) e^{i\omega t},$$

$$C(y, t) = \left(\frac{\sinh m_3 y}{\sinh m_3} \right) e^{i\omega t}.$$

The skin friction coefficient (τ) at the plate is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \left\{ \begin{aligned} & \frac{G_r}{m_1^2 - m_2^2} \left(\frac{m_2}{\sinh m_2} - \frac{m_1}{\sin m_1} \right) - \frac{G_c}{m_3^2 - m_2^2} \left(\frac{m_2}{\sinh m_2} - \frac{m_3}{\sinh m_3} \right) + \\ & \frac{\lambda}{m_2^2} \left(\frac{m_2}{\sinh m_2} (1 + e^{m_2}) \right) + \frac{\lambda}{m_2^2} ((1 + e^{m_2})) \end{aligned} \right\} e^{i\omega t}$$

The Local Nusselt number (Nu) is given by

$$Nu = \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \left(\frac{m_1}{\sin m_1} \right) e^{i\omega t}.$$

The Sherwood number (Sh) is given by

$$Sh = \left(\frac{\partial C}{\partial y} \right)_{y=0} = \left(\frac{m_3}{\sinh m_3} \right) e^{i\omega t}.$$

Results and discussions

The above mentioned numerical scheme is carried out for various values of physical parameters, namely, the Grashoff number(Gr), modified Grashoff number(Gc)the magnatic parameter (M), the Prandtl number (Pr), Permiability parameter porous medium(K) Heat source parameter(Q) and the suction/injection parameter (S), to obtain the effects of those parameters on dimensionless velocity, temperature and concentration distributions. The obtained computational results are presented graphically in Figures 1-13. Fig.1 exhibits the velocity profiles for several values of Grashoff parameter(Gr). It is seen that the velocity at any point of the fluid decreases when $t=0.1; Pe=1; w=1; E=0.5; N=2; sc=0.6; j=1; Gc=1; R=0.5; S=1; H=1$ when Grashoff parameter(Gr) increases. Fig.2 exhibits the velocity profiles for several values of Grashoff parameter(Gr) . It is seen that the velocity at any point decreases when $t=0.1; Pe=1; w=1; E=0.5; N=2; sc=0.6; j=1; Gr=1; R=0.5; S=1; H=1$ and when modified Grashoff parameter(Gc) increases. Fig.3 exhibits the velocity



profiles for several values of magnetic number (j). It is seen that the velocity at any point of the fluid decreases when $t=0.1; Pe=1; w=1; E=0.5; N=2; sc=0.6; Gr=1; Gr=1; R=0.5; S=1; H=1$ and when magnetic number (j) increases. Fig.4 exhibits the velocity profiles for several values of Schmidt number (Sc). It is seen that the velocity at any point of the fluid increases when $t=0.1; Pe=1; w=1; E=0.5; N=2; j=1; Gc=1; Gr=1; R=0.5; S=1; H=1$ and when Schmidt number (Sc) increases. Fig.5 exhibits the heat profile for several values of unsteadiness parameter (N). It is seen that the temperature at any point of the fluid when $j=5, E=0.5, N=2, sc=0.6, t=0, and w=1$ increases when unsteadiness parameter N increases. Fig.6 exhibits the heat profile for several values of pecklet unsteadiness parameter (Pe). It is seen that the temperature at any point of the fluid when $j=5, E=0.5, N=2, sc=0.6, t=0, and w=1$ increases when unsteadiness parameter (N) increases. Fig.7 exhibits the heat profile for several values of radiation parameter (E). It is seen that the temperature at any point of the fluid when $j=5, Pe=0.5, N=2, sc=0.6, t=0.1, and w=1$ increases when radiation parameter (E) increases.

Fig.8 exhibits the concentration profile for several values of radiation parameter (E). It is seen that the temperature at any point of the fluid when $j=5, Pe=0.5, N=2, sc=0.6, t=0.1, and w=1$ increases when radiation parameter (E) increases.

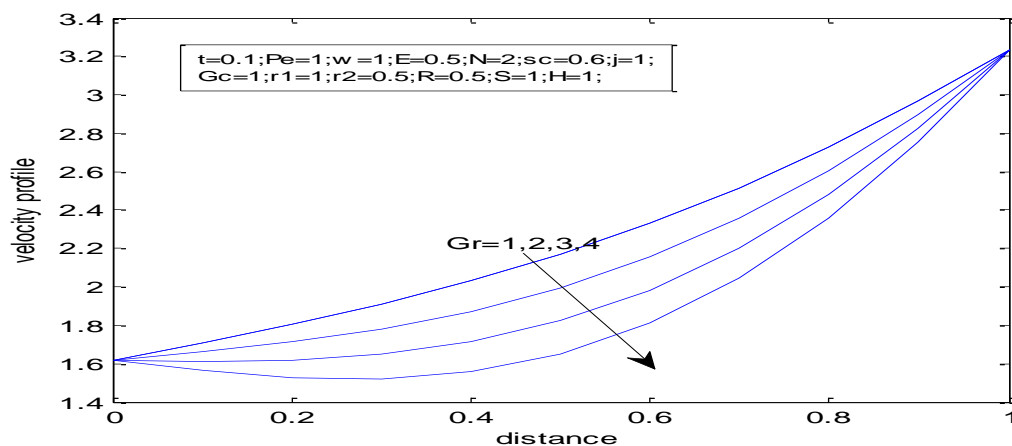


Figure 1 Effect of Gr on velocity

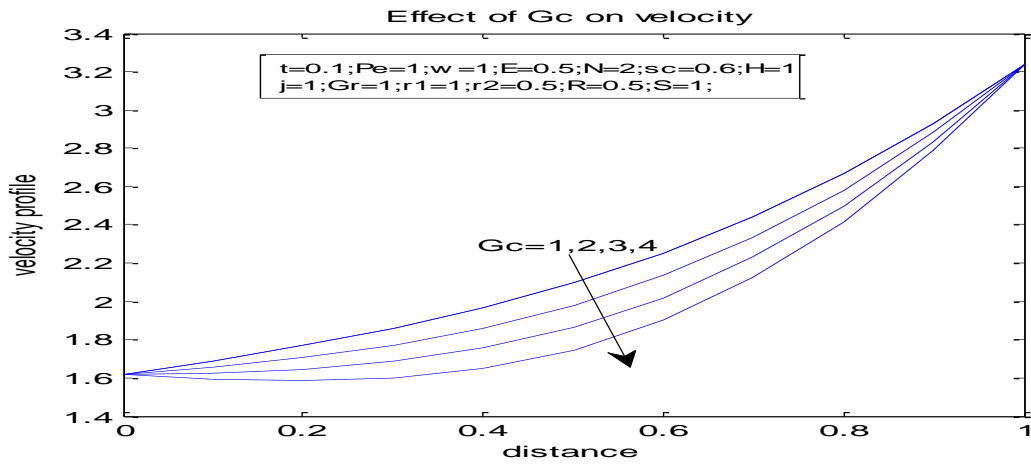


Figure 2 Effect of G_c on velocity profile

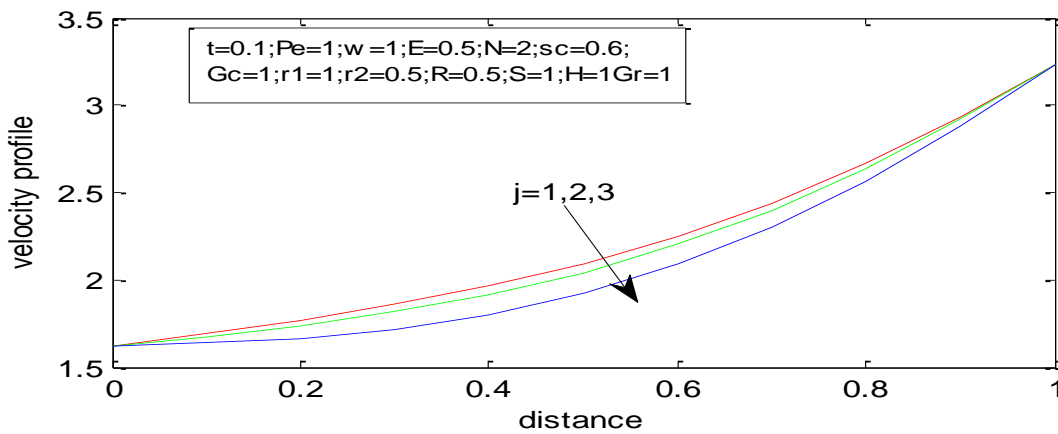


Figure 3 Effect of j on velocity

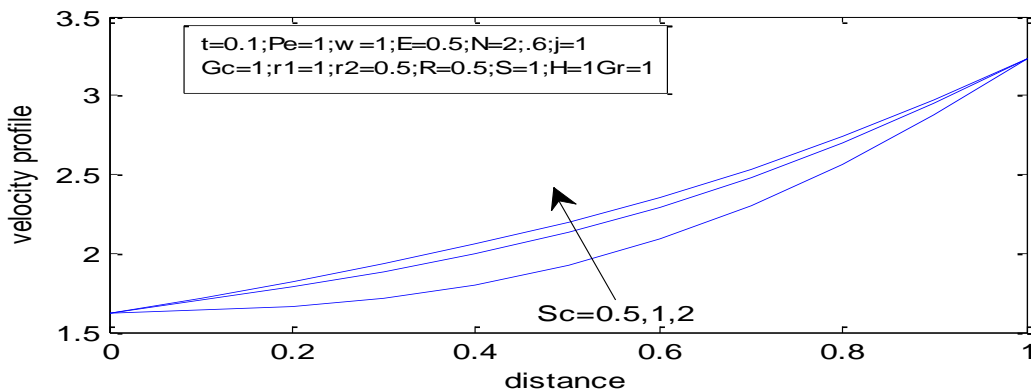


Figure 4 Effect of Sc on velocity

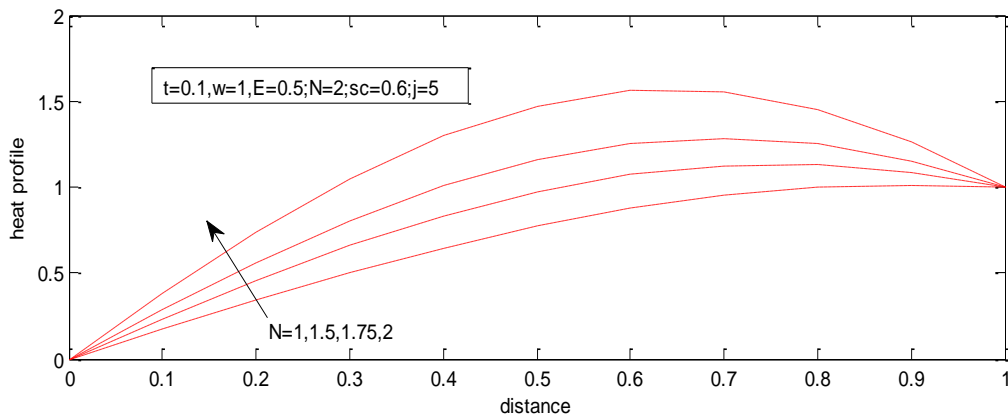


Figure 5 Effect of J on heat profile

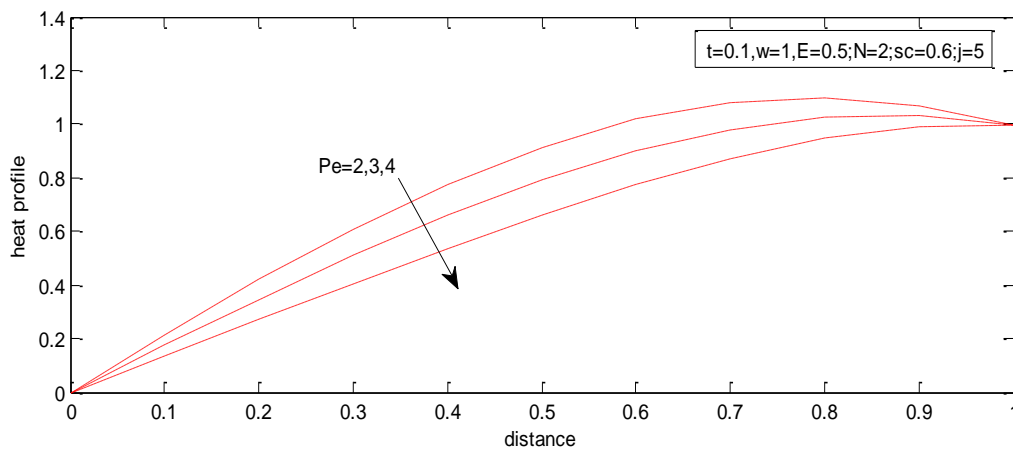


Figure 6 Effect of Pe on heat profile

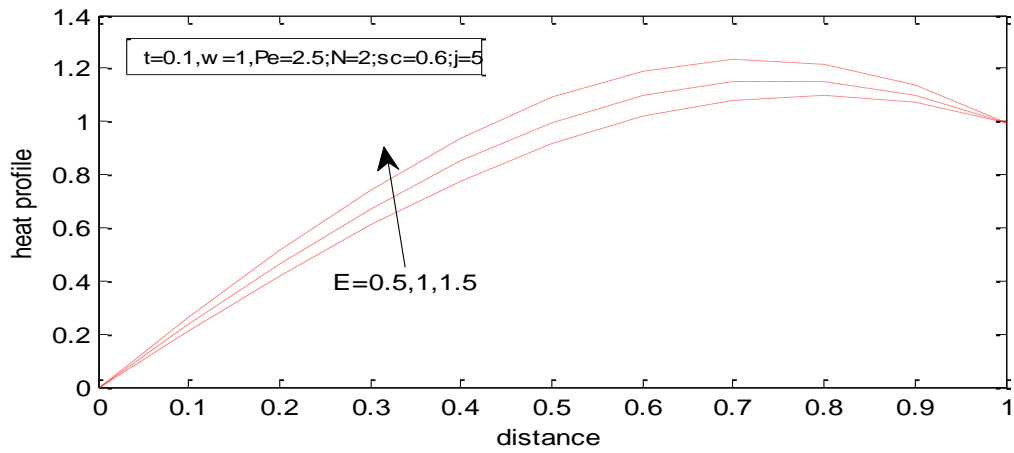


Figure 7 Effect of E on heat profile

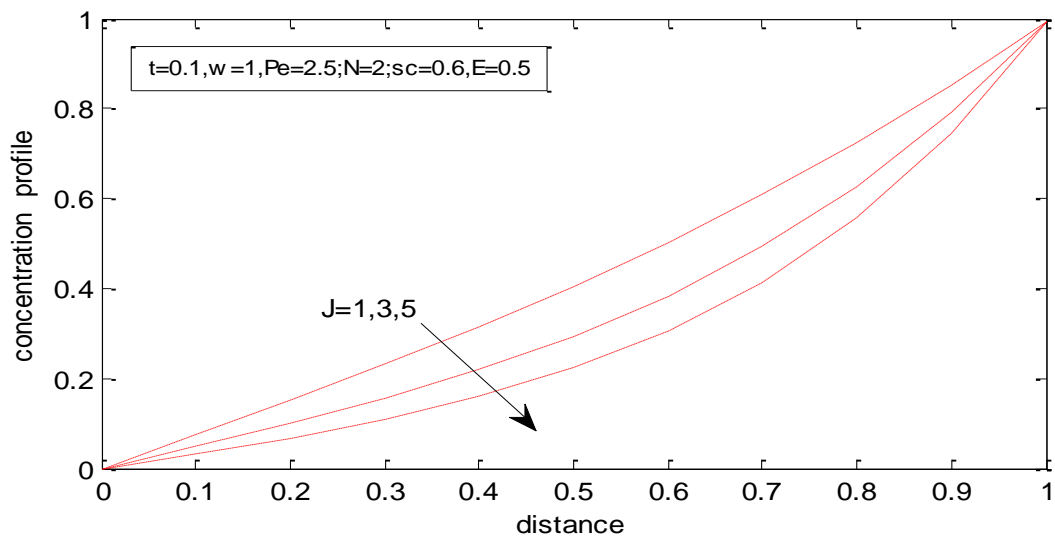


Figure 8 Effect of j on Concentration profile



Conclusions

In this study the sores and radiation effects on an unsteady MHD free convection nanofluid through porous medium with suction/injection and heat source in the presence of chemical reaction was investigated. The resulting governing equations were solved by the regular multiple perturbation technique. From the study, the following remarks can be summarized. Fluid velocity decreases with increasing Schmidt parameter (Sc) and modified while a reverse effect in the velocity distribution for Grashoff parameter (Gr), Modified Grashoff parameter (Gc), is observed. Fluid temperature decreases with increasing Peclet number (Pe), whereas a reverse effect is identified when increasing unsteadiness parameter (N) and Radiation parameter (E). The concentration profile decreases when increases chemical reaction parameter (J).

References

1. Raptis, And H. S. Takhar: "Heat Transfer From Flow Of An Elastico-Viscous Fluid", Int. comm. Heat and mass transfer, Vol. 16, pp. 193-917 (1989).
2. K. N. Mehta and K. N. Rao, "Buoyancy induced flow of non-Newtonian fluids in a porous medium past a vertical plate with non-uniform surface heat flux", Int. J. Eng. Sci., Vol 32, pp. 297-302 (1994).
3. J. S. Roy And N. K. Chaudhury: "Heat transfer by laminar flow of an elastico-viscous liquid along a plane wall with periodic suction", Czech. J. Physics, Vol. 30, pp. 1199-1209 (1980).
4. H. Veena, V. K. Pravin, K., And S. C. Padashetty: "Non-similar solutions in Viscoelastic mhd flow and heat transfer over a porous stretching sheet with internal heat generation and stress work", Int. J. Modern mathematics, Vol 2, pp. 267-281 (2007).
5. Elbashbeshy, E.M.A., Bazid, M.A.A., (2004), Heat transfer over an unsteady stretching surface, Heat Mass Transfer, Vol.41, pp.1-4.
6. Sharidan, S., Mahmood, T., Pop, I., (2006), Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet, Int J Appl Mech Eng., Vol.11, pp.647-54.
7. Swati Mukhopadhyay, Prativa Ranjan De, Krishnendu Bhattacharyya, G.C. Layek, (2013), Casson fluid flow over an unsteady stretching surface, Ain Shams Engineering Journal, Vol.4, pp.933-938.
8. ADash, R.K., Mehta, K.N., Jayaraman, G., (1996), Casson fluid flow in a pipe filled with a homogeneous porous medium, Int J Eng Sci., Vol.34(10), pp.1145-56.
9. Nadeem S, Ul Haq R, Lee C., (2012), MHD flow of a Casson fluid over an exponentially shrinking sheet, Sci Iran., Vol.19(6), pp.1550-3.
10. Boyd, J., Buick, J.M., Green, S., (2007), Analysis of the Casson and Carreau-Yasuda non-Newtonian blood models in steady and oscillatory flow using the lattice Boltzmann method, Phys Fluids, Vol.19, pp.93-103.
11. Mustafa, M., Hayat, T., Pop, I., Hendi, A., (2012), Stagnation-point flow and heat transfer of a Casson fluid towards a stretching sheet, Z Naturforsch, Vol.67a, pp.70-6.
12. J. G. Oldroyd: "On the formulation of rheological equations of state". Proc Roy Soc (Lond) Ser, A. Vol.200, No 1063, pp. 523-
13. J. Rao: "Flow of a Johnson-Segalman fluid between rotating coaxial cylinders with and without suction". Int. J. Nonlinear Mech, Vol. 34, pp. 63-70 (1999)



14. T. Hayat, M. Sajid, I. Pop: "Three-dimensional flow over a stretching surface in a viscoelastic fluid" *Nonlinear Analysis: Real World Applications*, Vol.9, pp. 1811-1822 (2008).
15. Ching-Ming Chiang, Kuo-Hsiang Chien, Han-Ming Chen, Chi-Chuan Wang "Theoretical study of oscillatory phenomena in a horizontal closed-loop pulsating heat pipe with asymmetrical arrayed mini channel" *International Communications in Heat and Mass Transfer*, Vol.39, pp. 923-930 (2012).
16. E. Scribber, D. Baird and P. Wapperom: "The role of transient rheology in polymeric sintering". *Rheologica Acta* Vol. 45, pp.825-839 (2006).
17. Z. Wouter, M. Hendriks and M.T. Hart: "A velocity-based approach to visco-elastic flow of rock". *Math. Geol.* Vol. 37, pp.141-162 (2005).
18. B. Sudhakar and S. Venkataramana: "MHD flow of visco elastic fluid past a permeable bed". *Reg. Engg. Heat Mass Transfer*.Vol.10, No.3, p-221-246 (1988).
19. R. C. Chaudhary, And P. Jain: "Hall effect on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation". *Theoretical and Applied Mechanics*, Vol. 33, pp. 281-309 (2006).
20. S. Gouse Mohiddin, V. R. Prasad, S. V. K. Varma, and O. Anwar Bég, Numerical study of unsteady free convective heat and mass transfer in a walters-B viscoelastic flow along a vertical cone, *Int. J. of Appl. Math and Mech.* 6 (15): 88-114, (2010)
21. Chamkha, A.J., Aly, A.M., Mansour, M.A., (2010), Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects, *Chem Eng Commun.*, Vol.197, pp.846-58.
22. K. R. Rajgopal: "On Stoke's problem for non-Newtonian fluid, *Acta Mech.* Vol.48, pp. 223-239 (1983).
23. K. D. Singh: "Exact solution of an oscillatory MHD flow in a channel filled with porous medium", *Int. J. of Applied Mechanics and Engineering*, Vol. 16, pp.277-283 (2011).
24. Makinde, O.D., and Aziz A., (2010), "MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition," *International Journal of Thermal Sciences*, Vol. 49, pp.1813-1820
25. Ajay Kumar Singh: "Heat source and radiation effects on magneto-convection flow of a viscoelastic fluid past a stretching sheet: Analysis with Kummer's functions" *International Communications in Heat and Mass Transfer*, Vol.35, pp. 637-642 (2008).
26. Mahmoud Bahmani, Mohammad Amin Khademi, Saheb Maghsoodloo Babakhani and DariushMowla, (2013), Theoretical Investigation of Power-Law Fluids Velocity Profile Between Two Parallel Plates, *World Applied Sciences Journal*, Vol.27 (10), pp.1355-1361.
27. Elbashbeshy, E.M.A., Bazid, M.A.A., (2004), Heat transfer over an unsteady stretching surface, *Heat Mass Transfer*, Vol.41, pp.1-4.
28. Sharidan, S., Mahmood, T., Pop, I., (2006), Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet, *Int J Appl Mech Eng.*, Vol.11, pp.647-54.
29. Hayat, T., Awais, M., (2011), Simultaneous effects of heat and mass transfer on time dependent flow over a stretching surface, *Int J Numer Meth Fluids*, Vol.67, pp.1341-57.
30. Hayat, T., Awais, M., Safdar, A., Hendi, A.A., (2012), Unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction. *Non-Linear Anal: Model Control*, Vol.17, pp.47-59.



31. Swati Mukhopadhyaya, M. Golam Arifb, and M. Wazed Ali Pkc, (2013), Effects of transpiration on unsteady MHD flow of an upper convected Maxwell (UCM) fluid passing through a stretching surface in the presence of a first order chemical reaction, Chin. Phys. B Vol. 22, No. 12, 124701.
32. Swati Mukhopadhyay, Prativa Ranjan De, Krishnendu Bhattacharyya, G.C. Layek, (2013), Casson fluid flow over an unsteady stretching surface, Ain Shams Engineering Journal, Vol.4, pp.933–938.
33. H. Pascal, “Rheological effects of non-Newtonian behavior of displacing fluids on stability of a moving interface in radial oil displacement mechanism in porous media”. Int. J. Eng. Sci. Vol. 24, pp. 1465–1476 (1986).
34. P. V. Satyanarayana: “Chemical reaction and thermal radiation effects on an steady mhd free convection flow past an infinite vertical plate with variable suction and heat source or sink” , IJMMSA. Vol. 4, pp. 27-45 (2011).
35. P. V. Satyanarayana, D.Ch. Kesavaiah and S. Venkataramana: “Viscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi- infinite vertical permeable moving porous plate”. International Journal of Mathematical Archive. Vol. 2, pp.476-487 (2011).
36. Andersson, H.I., Aarseth, J.B., Dandapat, B.S., (2000), Heat transfer in a liquid film on an unsteady stretching surface, Int J Heat Mass Transfer, Vol.43, pp.69–74.
37. Tsai, R., Huang, K.H., Huang, J.S., (2008), Flow and heat transfer over an unsteady stretching surface with a non-uniform heat source, Int Commun Heat Mass Transfer, Vol.35, pp.1340–3.
38. M. A. Hossain and H. S. Takhar: “Radiation effect on mixed convection along a vertical plate with uniform surface temperature”, Heat and Mass Transfer, Vol.31, pp. 243-248 (1996).
39. Eshetu Haile and Shankar, B., (2014), Heat and Mass Transfer in the Boundary Layer of Unsteady Viscous Nanofluid along a Vertical Stretching Sheet, Hindawi Publishing Corporation Journal of Computational Engineering, Volume 2014, Article ID 345153, 17 pages.
40. G. A. Vlastos: “The viscoelastic behavior of blood and blood-like model fluids with emphasis on the superposition of steady and oscillatory shear”. Clinical.Hemorheology and Microcirculation. Vol. 19, pp. 177–179 (1998).
41. J.A. Tichy: “Non-Newtonian lubrication with the convected Maxwell model”.ASMEJ. Tribol. Vol. 118, pp. 344–348 (1996).
42. Mustafa, M., Hayat, T., Pop, I., Hendi, A., (2012), Stagnation-point flow and heat transfer of a Casson fluid towards a stretching sheet, Z Naturforsch, Vol.67a, pp.70–6.