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Some mappings on smooth fuzzy topological and bitopological spaces

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Abstract:Smooth fuzzy topologies are an extension of both, crisp topologies and fuzzy topologies, where not only the sets but also the axioms are fuzzified. In this paper, we study some types of smooth functions on smooth fuzzy topological spaces, in the sense of Sostak and Ramadan. The relations among various types of continuity of smooth fuzzy functions on a fuzzy set and at fuzzy point belonging to the fuzzy set are discussed. Further we extend these notions to pairwise smooth mappings between smooth fuzzy bitopological spaces. Also various types of open, closed, continuous mappings are introduced and their interrelations are studied.

Keywords: Fuzzy topology, Smooth fuzzy topology, Smooth fuzzy bitopology, Pairwise continuous map, Pairwise open map

Mathematics Subject Classification: 54A40, 03E72

1. Introduction

The concept of fuzzy sets introduced by Zadeh [8] has found wide applications in various branches of Mathematics . in 1968,C.L.Chang[6] introduced the theory of fuzzy topological spaces and since then various notions in classical topology have been extended to fuzzy topological spaces .Sostak[5]introduced the fundamental concept of a fuzzy topological structure as an extension of both crisp topology and fuzzy topology in the sense that not only objects were fuzzified but also the axiomatics.In[3]Ramadan gave a similar definition namely 'smooth fuzzy topological spaces 'for lattice L= [0,1] ,it has been developed in many directions .In 1989,Kandil[4] introduced the concept of fuzzy bitopological spaces. In this paper ,we extend the notion of smooth mappings between two smooth topological spaces,to smooth mappings between two smooth fuzzy bitopological spaces.

2. Prelimanaries:

The class of all fuzzy sets on a universal set X will be denoted by L^X where L is a completely distributive lattice with order reversing involution.

Definition 2.1: A fuzzy point x_α for $\alpha \in L$ is an element of L^X such that

$$x_\alpha(y) = \begin{cases} \alpha \; if \; y = x \\ 0 \; if \; y \neq x \end{cases}$$

The set of all fuzzy points in X is denoted by Pt(X).

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A fuzzy point $x_\alpha \in A$ if and only if $\leq A(x) \quad \forall x \in X$.

Definition 2.2: A mapping $\tau: L^X \to L$ is called a smooth L-fuzzy topology on X if it satisfies the following conditions

i) $\tau(0) = \tau(1) = 1$
ii) $\tau(\eta_1 \wedge \eta_2) \geq \tau(\eta_1) \wedge \tau(\eta_2)$ $for\ any\ \eta_1, \eta_2 \in L^X$
iii) $\tau(\vee_{i\in I} \eta_i) \geq \wedge_{i\in I} \tau(\eta_i)$ $for\ any\ \{\eta_i : i \in I\}\ in\ L^X$.

The pair (X, τ) is called a smooth L-fuzzy topological space.
For every $\eta \in L^X$, $\tau(\eta)$ is called the degree of openness of η .

Note that: A smooth L-fuzzy topology on X is a fuzzy subset of L^X while a L-fuzzy topology on X is a crisp subset of L^X.

Theorem 2.1: Let (X, τ) be a smooth fuzzy topological space then for each $\alpha \in L\backslash\{0\}$

$$\tau_\alpha = \{\eta \in L^X | \tau(\eta) \geq \alpha\}$$

is a L-fuzzy topology on X.

Definition 2.3: Let (X, τ) be a smooth l-fuzzy topological space we define $\mathcal{F}: L^X \to L$ as $\mathcal{F}(\eta) = \tau(\eta')$ for every $\eta \in L^X$, then $\mathcal{F}$ has the following properties

i) $\mathcal{F}(0) = \mathcal{F}(1) = 1$
ii) $\mathcal{F}(\eta_1 \vee \eta_2) \geq \mathcal{F}(\eta_1) \wedge \mathcal{F}(\eta_2)$ $for\ any\ \eta_1, \eta_2 \in L^X$
iii) $\mathcal{F}(\wedge_{i\in I} \eta_i) \geq \wedge_{i\in I} \mathcal{F}(\eta_i)$ $for\ any\ \{\eta_i : i \in I\}\ in\ L^X$.

$\mathcal{F}(\eta)$ is called the degree of closedness of η .
We call (X, $\mathcal{F}$) as a smooth L-fuzzy cotopological space.
Definition 2.4: Let (X, τ) be a smooth L-fuzzy topological space and $\eta \in L^X$.
The smooth interior of η, denoted as η° is defined as

$$\eta^\circ = \vee \{\mu \in L^X | \mu \leq \eta\ and\ \tau(\mu) > 0\}$$

Similarly,the smooth closure of η, denoted as $\bar{\eta}$,is defined as
$\bar{\eta} = \wedge \{\mu \in L^X \ | \mu \geq \eta\ and\ \tau(\mu') > 0\}$
Theorem 2.2 : Let (X, τ) be a smooth L-fuzzy topological space and $\mu, \eta \in L^X$ then

i) $\mu \leq \eta \Rightarrow \mu^\circ \leq \eta^\circ\ and\ \bar{\mu} \leq \bar{\eta}$
ii) $(\bar{\mu})' = (\mu')^\circ$, $(\mu^\circ)' = (\overline{\mu'})$
iii) $\tau(\mu) > 0 \Rightarrow \mu = \mu^\circ$
iv) $\mathcal{F}(\mu) > 0 \Rightarrow \mu = \bar{\mu}$

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3. Smooth Functions on smooth L-fuzzy topological spaces

Definition 3.1: Let (X,τ_1) and (Y, τ_2) be smooth L-fuzzy topological spaces .The function $f: X \to Y$ is called smooth fuzzy continuous iff $\tau_1(f^{-1}(\mu)) \geq \tau_2(\mu)$ for all $\mu \in L^Y$

Theorem 3.1: If $f: X \to Y$ is a smooth fuzzy continuous function then for every closed set η , $f^{-1}(\eta)$ is a closed fuzzy set in τ_2 .
Proof : Let η be a closed set in τ_2 .
Then, η' is open set in τ_2 .
$\therefore\ \tau_2(\eta') > 0$.
Since f is smooth fuzzy continuous on X , $\tau_1(f^{-1}(\eta')) \geq \tau_2(\eta')$

$$\therefore \tau_1\left(f^{-1}(\eta')\right) > 0$$

But, $f^{-1}(\eta')$ = X - $f^{-1}(\eta)$

$$\therefore \tau_1\left(f^{-1}(\eta')\right) = \tau_1(X - f^{-1}(\eta)) = 1 \wedge \tau_1(f^{-1}(\eta))' > 0$$

$$\therefore \tau_1(f^{-1}(\eta))' > 0$$

$\therefore f^{-1}(\eta)$ is closed.
Theorem3.2: Let (X_i, τ_i) be smooth L-fuzzzy topological spaces for i = 1,2,3. If f : $X_1 \to X_2$ and g: $X_2 \to X_3$ are smooth continuous maps then (g°f) is a smooth continuous map.
Proof: *Proof is straightforward.*
$\tau_1((g°f)^{-1}(\eta))= \tau_1\left(f^{-1}(g^{-1}(\eta))\right)$
$\geq \tau_2\ (g^{-1}(\eta)) \geq \tau_3\ (\eta)$.

Theorem 3.3 : Let (X,τ_X) and (Y,τ_Y) be smooth L-fuzzy subspaces, $f: X \to Y$ a smooth fuzzy continuous map ,A ⊂ X .Then f$|_A$: (A, τ_A) → (Y, τ_Y) is also smooth continuous.
Proof:

$$\tau_X|_A\ ((f|_A)^{-1}\ (\eta)) = \vee \{\tau_X(\lambda): \lambda \in L^Y, \lambda|_A = (f|_A)^{-1}(\eta)\},$$
$$\geq\ \tau_X(f^{-1}(\eta))$$
$$\geq \tau_Y(\eta)$$

Definition 3.2: Let (X,τ_1) and (Y, τ_2) be smooth L-fuzzy topological spaces .The function $f: X \to Y$ is called weakly smooth fuzzy continuous iff $\tau_1(f^{-1}(\mu)) > 0\ whenever\ \tau_2(\mu) > 0$ for $\mu \in L^Y$.
To discuss the continuity pointwise in the context of slfts ,we introduce smooth fuzzy continuity and weakly smooth fuzzy continuity at a fuzzy point.

Definition 3.3 : Let (X,τ_1) and (Y,τ_2) be smooth L-fuzzy topological spaces and f:(X,τ_1)→(Y,τ_2) be a fuzzy function. Then,f is said to be smooth fuzzy continuous at a fuzzy point $x_\alpha \in X$ if for every $V \leq Y$ with $f(x_\alpha) \in V$,there exists $U \leq X$ such that $x_\alpha \in U, f(U) \leq V\ and\ \tau_1(U) \geq\ \tau_2(V)$.

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Similarly, f is said to be weakly smooth fuzzy continuous at a point $x_\alpha \in X$ if for every $V \leq Y$ with $\tau_2(V) > 0$ and $f(x_\alpha) \in V$,there exists $U \leq X$ such that $\tau_1 \ (U) > 0, x_\alpha \in U, f(U) \leq V$.

Theorem 3.4: Let (X,τ_1) and (Y,τ_2) be smooth L-fuzzy spaces and f:$(X,\tau_1)\rightarrow(Y,\tau_2)$ be a fuzzy function.Then f is smooth continuous on X if and only if f is smooth continuous at every point $x_\alpha \in X$.
Proof: Suppose f is smooth continuous on X.
Let $x_\alpha \in X, V \leq Y$ *with* $f(x_\alpha) = t_\alpha \in V$ where t is such that f(x,t) = X(x).
Taking $U = f^{-1}(V)$, we have $\tau_1(U) \geq \tau_2(V)$ and $f(U) = f\left(f^{-1}(V)\right) \leq V$
Since α < X(x), α< V(t) we have U(x) =$f^{-1}(V)(x) = X(x) \wedge V(t) > \alpha$
So , $x_\alpha \in U$.
Hence, f is smooth L-fuzzy continuous at every point of X.
Conversely, suppose f is smooth fuzzy continuous at every point $x_\alpha \in X$.
Let $V \leq Y$
For every $x_\alpha \in f^{-1}(V)$ there is $U_{x_\alpha} \leq X$ such that
$x_\alpha \in U_{x_\alpha}$ and f(U_{x_α}) $\leq V$ *and* $\tau_1\left(U_{x_\alpha}\right) \geq \tau_2(V)$
Then clearly, $f^{-1}(V) = \vee U_{x_\alpha}$
and $\tau_1\left(f^{-1}(V)\right) = \tau_1\left(\vee U_{x_\alpha}\right) \geq \tau_1\left(U_{x_\alpha}\right) \geq \tau_2(V)$
∴ f is smooth fuzzy continuous on X.

Theorem 3.5: Let (X,τ_1) and (Y,τ_2) be smooth L-fuzzy spaces and f:$(X,\tau_1)\rightarrow(Y,\tau_2)$ be a fuzzy function.Then f is weakly smooth continuous on X if and then f is weakly smooth continuous at every point $x_\alpha \in X$.
Proof : Suppose that f is weakly smooth fuzzy continuous on X.
Let $x_\alpha \in X, V \leq Y$ *with* $\tau_2(V) > 0$ *and* $f(x_\alpha) \in V$.
If $U = f^{-1}(V)$, then $\tau_1(U) = \tau_1\left(f^{-1}(V)\right) > 0$ and $x_\alpha \in U$
Hence, f is smooth L-fuzzy continuous at every point of X.
Note that : There exists a weakly smooth fuzzy continuous function at every point of a fuzzy set X which is not weakly smooth fuzzy continuous on X.
Definition 3.4 : Let (X,τ_1) and (Y, τ_2) be smooth L-fuzzy topological spaces .The function $f: X \rightarrow Y$ is called α-weakly smooth fuzzy continuous iff $\tau_1\left(f^{-1}(\mu)\right) \geq \alpha$ *whenever* $\tau_2(\mu) \geq \alpha$ for $\mu \in L^Y$.
Theorem 3.6: If f is α-weakly weakly smooth fuzzy continuous $\forall \alpha \in (0,1]$ on X then f is weakly smooth continuous on x.
Proof: Let V≤Y with $\tau_2(V) > 0$
Choose α such that $0 < \alpha \leq \tau_2(V)$
Since f is weakly smooth fuzzy continuous
$\tau_1(f^{-1}(V)) \geq \alpha > 0$
and hence f is weakly smooth fuzzy continuous on X.
However, converse of above theorem is not true .

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There exist function f which is weakly smooth fuzzy continuous but not α-weakly smooth fuzzy continuous for some $\alpha \in (0,1]$

Definition 3.5: Let (X, τ_1) and (Y, τ_2) be smooth L-fuzzy topological spaces . A mapping f: $(X,\tau_1) \to (Y,\tau_2)$ is called smooth L-fuzzy open iff $\tau_2\big(f(\lambda)\big) \geq \tau_1(\lambda) > 0$ for every $\lambda \epsilon L^X$
A mapping f: $(X,\tau_1) \to (Y,\tau_2)$ is called smooth L-fuzzy closed iff $\tau_2\Big(f(\lambda')\Big) \geq \tau_1(\lambda') > 0$ for every $\lambda \epsilon L^X$
.

4. Smooth mappings between smooth L-fuzzy bitopological spaces:

Definition 4.1: A smooth L-fuzzy bitopological space (slfbts) is a triple (X, τ_1, τ_2) where τ_1 and τ_2 are arbitrary smooth L-fuzzy topologies on X.

Definition 4.2: Let (X, τ_1, τ_2) and (Y, $\mathcal{s}_1, \mathcal{s}_2$) be any two slfbts. A function f : (X, τ_1, τ_2) → (Y, $\mathcal{s}_1, \mathcal{s}_2$) is pairwise smooth fuzzy continuous if $\tau_i\big(f^{-1}(\mu)\big) \geq \mathcal{s}_i(\mu)\ for\ each\ \mu \in L^Y and\ i = 1,2.$

Note that: If we put $\tau_1 = \tau_2 = \tau$ and $\mathcal{s}_1 = \mathcal{s}_2 = \mathcal{s}$, then the definition of smooth fuzzy pairwise continuous map reduces to the corresponding definition of smooth fuzzy continuous map.

Definition 4.2: Let (X, τ_1, τ_2) and (Y, $\mathcal{s}_1, \mathcal{s}_2$) be any two slfbts. A function f : (X, τ_1, τ_2) → (Y, $\mathcal{s}_1, \mathcal{s}_2$) is weakly smooth pairwise fuzzy continuous if $\tau_i\big(f^{-1}(\mu)\big) > 0\ whenever\ \ \mathcal{s}_i(\mu) > 0\ for\ each\ \mu \in L^Y and\ i = 1,2.$

Definition 4.3: Let (X, τ_1, τ_2) and (Y, $\mathcal{s}_1, \mathcal{s}_2$) be any two slfbts. A function f : (X, τ_1, τ_2) → (Y, $\mathcal{s}_1, \mathcal{s}_2$) is called α- pairwise smooth fuzzy continuous if $\tau_i\big(f^{-1}(\mu)\big) \geq \alpha\ for\ \mu \in L^Y and\ \mathcal{s}_i(\mu) \geq \alpha\ , i = 1,2$

Definition 4.4: Let (X, τ_1, τ_2) and (Y, $\mathcal{s}_1, \mathcal{s}_2$) be any two slfbts. A function f : (X, τ_1, τ_2) → (Y, $\mathcal{s}_1, \mathcal{s}_2$) is somewhat pairwise smooth fuzzy continuous if $\tau_1\big(f^{-1}(\mu)\big) \geq \alpha\ or\ \tau_2(f^{-1}(V)) \geq \alpha\ for\ \mu \in L^Y and\ \mathcal{s}_1(\mu) \geq \alpha\ or\ \mathcal{s}_2(\mu) \geq \alpha, i = 1,2.$

Definition 4.5: Let (X, τ_1, τ_2) be any slfbts .$\lambda \in L^X$ is called pairwise dense smooth L-fuzzy set if there exists no $\mu \in L^X\ with\ \tau_1(\mu') > 0\ or\ \tau_2(\mu') > 0\ and\ \lambda < \mu < 1.$

Theorem 4.1: Suppose (X, τ_1, τ_2)and (Y, σ_1, σ_2) are any two slfbts and f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a function. Then f is smooth fuzzy continuous if and only if for every smooth L-fuzzy pairwise dense set $\lambda \in L^X$, f(λ) $\in L^Y$ is also smooth L-fuzzy pairwise dense set.
Proof: Suppose f is smooth fuzzy continuous and λ is a pairwise dense set in L^X.
If f (λ) ∈ L^Y is not smooth L-fuzzy pairwise dense set, then there exists $\nu \in L^Y$
such that $f\ (\lambda) < \nu < 1$ and $\sigma_i(\upsilon) > 0$ for $i = 1, 2.$

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Let $\sigma_1(v) > 0$. Since f is smooth fuzzy continuous $\tau_1(f^{-1}(v)) > \sigma_1(v) > 0$.
Also $\lambda < f^{-1}(v) < 1$, which is a contradiction.
So, $f(\lambda) \in L^Y$ is smooth L-fuzzy pairwise dense set.
Similarly, we can prove the converse.

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