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Heat Transfer for Slug Flow in Circular Tube

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Abstract

In this paper, the applicable non-linear equation in cylindrical co-ordinates representing the momentum and energy equations of combined dynamic and thermal boundary layers for Slug flow in circular tube have been reduced to simple form with the application of all the requisite boundary conditions. The resulting momentum equation is solved with the use of continuity equation to obtain the velocity profile, which is then used to obtain the temperature distribution adopting trial and error solution method. Heat transfer co-efficient for the associated problem has been calculated. The results thus obtained have been compared with the result calculated with the use of proposed empirical formulae of liquid metals

Key Words: Thermal boundary layer, Slug flow, Nusselt number, Prandtl number, Pecklet number

Introduction

Heat transfer associated with fluid flowproblem through a cylindrical tube is complicated. The result of such problems are widely used in analysis and design of heat exchangers [8,9,10,11]. For development of its technology, further study of the subject is needed. The turbulent duct flow in which the heat transport by turbulent mixing be small in comparison to conductive transport, is known as Slug flow. A region of slug flow is generated when a boiling liquid flows downward in a vertical tube where individual large bubbles (slugs) are separated from one another by layer of vapour-liquid emulsion.

Turbulent duct flow of liquid metals in which the heat transport by turbulent mixing is comparably small with that of the conductive transport, is slug flow type. Exact solution for laminar velocity and temperature profiles in a tube at a great distance from the entrance has been obtained by Gratz [5] and Nusselt [6] for content wall temperature. Eckert [7] obtained approximate solution of the same problem. Callender [8] and Nusselt [6] obtained heat transfer to the walls of a tube by solving the concerning differential equations. Gratz [5] obtained solution for slug flow through tube. Gupta and Mishra [9] obtained solution for the problem of heat transfer in liquid metals. Recently, Walsh et.al. [12] shows that heat transfer characteristics in the entrance region of slug flows are governed by a combination of conventional single phase and plug flow behaviours with short slugs approaching the theoretical plug limit and long slugs approaching the single phase flow limit.

In this paper the non-linear differential equations in cylindrical co-ordinates representing the combined dynamic and thermal boundary layers for slug flow in circular tube have been reduced to simpler form with the applications of all the required boundary conditions. On solving the resulting momentum equation with the use of continuity equation a velocity profile is obtained. The same profile is then used to solve the energy equation and a temperature profile has been obtained. A mathematical relation is also been developed to predict the heat transfer rate. Comparison of these results has also been made with that of the calculated results obtained with the use of the proposed empirical formulae of liquid-metals performing duct flow when the heat transport by turbulent mixing is negligibly small.

Problem Statement

Consider a steady viscous incompressible fluid flow through a uniform circular tube. Let x-axis be along the generator of the tube having radius r_o. Let the axial, radial and azimuthal co-ordinates be x,r and θ and u,v,w be the velocities along the respective axes.

Assumptions

1. All body forces are negligible.
2. The fluid is forced through the tube by external means and is independent of the temperature field in the fluid.

Governing Equations and different conditions

The non-linear equations in cylindrical co-ordinates (x, r, θ) representing the flow are

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}+\frac{w}{r}\frac{\partial u}{\partial \theta}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial x^2}\right] \qquad (1)$$

$$\rho\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}+\frac{w}{r}\frac{\partial v}{\partial \theta}-\frac{w^2}{r}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{v}{r^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2}{r}\frac{\partial v}{\partial x}+\frac{\partial^2 v}{\partial x^2}\right] \qquad (2)$$

$$\rho\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial r}+v\frac{w}{r}+\frac{w}{r}\frac{\partial w}{\partial t}\right]=$$
$$-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v}{\partial \theta}+\frac{\partial^2 w}{\partial x^2}\right] \qquad (3)$$

Equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{1}{r}\frac{\partial w}{\partial \theta}=0 \qquad (4)$$

Equation of energy is

$$\rho C_p\left[u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial r}+\frac{w}{r}\frac{\partial t}{\partial \theta}\right]=K\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \theta^2}+\frac{\partial^2 t}{\partial x^2}\right]$$
$$+\left[2\left\{\left(\frac{\partial \theta}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2\right\}\right.$$
$$+\left\{r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)+\frac{1}{r}\frac{\partial v}{\partial \theta}\right\}^2+\left(\frac{1}{r}\frac{\partial w}{\partial \theta}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial w}{\partial r}\right)^2+\cdots \qquad (5)$$

where ρ is the density of the fluid, p is pressure, μ is viscosity, C_P is heat capacity, K is thermal conductivity.

Conditions for laminar flow through tube are

$$w=0\,,\quad \frac{\partial}{\partial \theta}=0 \qquad (6)$$

Condition for rotational symmetry is

$$\frac{\partial v}{\partial x}=0 \qquad (7)$$

Conditions for uniform pressure over a cross-section is

$$\frac{\partial p}{\partial r} = 0 \tag{8}$$

We find that V will be uniform if condition for uniform velocity as required in Slug flow is

$$\frac{\partial p}{\partial x} = 0 = \frac{\partial v}{\partial x} \tag{9}$$

Solution of Momentum and Energy equations:

Under conditions (6) to (8) and (9), equation (1) reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0 \tag{10}$$

With boundary conditions

$$\text{For} 0 \leq r \leq r_{0\,;}\ \frac{\partial u}{\partial r} = 0\ ;\mathrm{V} = 0 \tag{11}$$

Where r_0 is radius of the tube. Integrating equation (10), we get

$$\frac{\partial u}{\partial r} = \frac{A_1}{r} \tag{12}$$

Application of (11) reduces it to

$$\frac{A_1}{r} = 0\ ; or, A_1 = 0$$

i.e. $\frac{\partial u}{\partial r} = 0$

or, u = constant (13)

Under conditions (6) to (8), equation (5) reduces to

$$\frac{\rho C_p v}{K}\frac{\partial t}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{14}$$

with boundary conditions

$$r = 0\ ; \frac{\partial t}{\partial r} = 0$$

$$r = r_0\ ;\ t = t_0 \tag{15}$$

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The heat transfer co-efficient for flow in tubes is defined in terms of mean temperature of the fluid. Mean temperaturet_m, is defined as

$$t_m = \frac{\int_0^{r0} ut\,2\pi r\ dr}{\int_0^{r0} u2\pi r\ dr} \tag{16}$$

And heat transfer coefficient "h" as

$$h = \frac{q\ /A}{t_0 - t_m} = -\frac{K}{r_0}\frac{\partial}{\partial\left(\frac{r}{r_0}\right)}\left(\frac{t_0 - t}{t_0 - t_m}\right) = \frac{K\left(\frac{\partial t}{\partial r}\right)_{r=r_0}}{t_0 - t_m} \tag{17}$$

Where q = rate of heat transfer, and A = cross-sectional area. We define a fully developed temperature profile to exist when $\left(\frac{t_0-t}{t_0-t_m}\right)$ is a unique function of $\frac{r}{r_0}$ independent of x, then

$$\frac{t_0 - t}{t_0 - t_m} = f\left(\frac{r}{r_0}\right)$$

or $$\frac{\partial}{\partial x}\left(\frac{t_0 - t}{t_0 - t_m}\right) = 0 \tag{18}$$

or $$\left(\frac{\partial t_0}{\partial x} - \frac{\partial t}{\partial x}\right) - \left(\frac{t_0 - t}{t_0 - t_m}\right)\left(\frac{\partial t_0}{\partial x} - \frac{\partial t_m}{\partial x}\right) = 0 \tag{19}$$

Solution of energy equation (14):

Case 1: **when (q/A) is uniform**

Since q/A and h are uniform, $(t_0 - t_m)$ is also uniform along the pipe, then

$$\frac{\partial t_0}{\partial x} = \frac{\partial t_m}{\partial x}$$

and from equation (19)

$$\frac{\partial t}{\partial x} = \frac{\partial t_0}{\partial x} = \frac{\partial t_m}{\partial x}\text{, independent of r} \tag{20}$$

On the basis of equation (13) and (20), the term on left hand side of equation (14)can be treated as constant. Integrating equation (14) w.r.t. r we get,

$$r\frac{\partial t}{\partial r} = \frac{\rho C_P u}{2K}\frac{\partial t_m}{\partial x}r^2 + c_1 \tag{21}$$

Under condition (15), we find

$$c_1 = 0$$

Substituting $c_1 = 0$ in (21) and then integrating w.r.t. r, we get

$$t = \frac{1}{4}\frac{\rho C_P u}{K}\frac{\partial t_m}{\partial x} r^2 + c_2 \tag{22}$$

Using equation (15) in it, we get

$$t_0 = \frac{1}{4}\frac{\rho C_P u}{K}\frac{\partial t_m}{\partial x} r_0^2 + c_2 \tag{23}$$

Subtraction of equation (22) from equation (23) gives

$$t_0 - t = \frac{\rho C_P u}{4K}\frac{\partial t_m}{\partial x}(r_0^2 - r^2) \tag{24}$$

Substituting value of r, obtaining from equation (24) in equation (16), we ultimately get

$$t_0 - t_m = \frac{\int_0^r u(t_0 - t)2\pi\, r\, dr}{\int_0^r 4\, 2\pi\, r\, dr} \tag{25}$$

Substituting value of u obtaining from equation (13) and that of $(t_0 - t)$ from (24) in (25) and the performing the indicated integration, we get $(t_0 - t_m)$as ,

$$t_0 - t_m = \frac{1}{4}\frac{\partial C_P}{K}\left(\frac{r_0^2}{2}\right)\frac{\partial t_m}{\partial x} \tag{26}$$

Value of $\left(\frac{\partial t}{\partial r}\right)_{r=r_0}$ is obtained from equation (24) as

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_0} = \frac{1}{2}\frac{\sigma C_P}{K}\frac{\partial t_m}{\partial x} \tag{27}$$

Substituting value of $(t_0 - t_m)$ and $\left(\frac{\partial t}{\partial r}\right)_{r=r_0}$ obtained from equations (26) and (27) in equation (17), we get value of "h" as

$$h = \frac{4K}{r_0} = \frac{8K}{D_e}$$

where D_e is the hydraulic diameter of the tube.

$$Nu_{D_e} = \frac{h\, D_e}{K} \tag{28}$$

$$= 8$$

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Division of equation (24) by equation (26) gives

$$\frac{t_0-t}{t_0-t_m} = 2\left(1-\frac{r^2}{r_0^2}\right) \tag{29}$$

Case II : when t_0 is uniform equation

Since $t_0$is uniform, so$\frac{\partial t_0}{\partial x} = 0$ .Substituting it in equation (19), we get

$$\frac{\partial t}{\partial x} = \left(\frac{t_0-t}{t_0-t_m}\right)\frac{\partial t_m}{\partial x} \tag{30}$$

Which is dependent on radial position in the pipe. Substituting value of $\frac{\partial t}{\partial x}$,obtained from equation (30), in equation (14) we get

$$u\left(\frac{t_0-t}{t_0-t_m}\right)\frac{\partial t_m}{\partial x} = \frac{K}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)\right] \tag{31}$$

Substituting value of $\left(\frac{t_0-t}{t_0-t_m}\right)$ obtained from equation (29) in equation (31), we get

$$2u\frac{\partial t_m}{\partial x} = \left(1-\frac{r^2}{r_0^2}\right) = \frac{K}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)\right]$$

Integration of this using equation (13), gives a new temperature distribution in the form

$$\frac{t_0-t}{t_0-t_m} = \frac{3}{4}\left[3r_0^4 - 4r_0^2r^2 + r^4\right] \tag{32}$$

Substituting value of $\left(\frac{t_0-t}{t_0-t_m}\right)$ obtained from equation (32) in equation (31) again, we get

$$\frac{3}{4}\frac{\rho C_p}{K}u\frac{\partial t_m}{\partial x}\left[3r_0^4 - 4r_0^2r^2 + r^4\right] = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)$$

On integrating this, we ultimately get

$$t_0 - t = \frac{3}{4}\frac{\rho C_p}{K}u\frac{\partial t_m}{\partial x}\left[\frac{19}{36}r_0^6 - \frac{3}{4}r_0^4r^2 + \frac{1}{4}r_0^2r^4 - \frac{1}{36}r^6\right] \tag{33}$$

Substituting the value of $(r_0 - t)$ and u obtained from equations (33) and (13) in equation (25), and after performing the indicated integration, we get,

$$t_0 - t_m = \frac{3}{4}\frac{\rho C_p}{K}u\frac{\partial t_m}{\partial x}.\frac{11}{48}r_0^6 \tag{34}$$

Differentiation of equation (34) gives $\left(\frac{\partial t}{\partial r}\right)_{r=r_0}$ as,

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_0} = \frac{1}{2}\frac{\rho C_p}{K} u \frac{\partial t_m}{\partial x} r_0^5 \quad (35)$$

The value of $(t_0 - t_m)$ and $\left(\frac{\partial t}{\partial r}\right)_{r=r_0}$ obtained from equations (34) and (35) are substituted in equation (17) and the value of "h" is obtained as

$$h = \frac{32\ K}{11\ r_0}$$

or, $$h = \frac{64\ K}{11\ D_e}$$

Thus, $Nu_{D_e} = 5.818$

The value of Nu_{D_e} for the cases q/A and to uniform, are found by the earlier researchers as Nu_{D_e} 8.00 and 5.75 respectively.

Lyon(6) suggested an approximate formula for predicting the Nusselt number for the range of Prandtl numbers characteristics of liquid metals performing turbulent flow in a straight circular tube as

$$Nu_{D_e} = 7 + 0.025\ Pe_{D_e} 0.8 \quad (36)$$

when (q/A) is uniform.

For uniform (t_0), Seban and Shimazaki (7) suggested formula of the form

$$Nu_{D_e} = 5 + 0.025\ Pe_D\ 0.8 \quad (37)$$

Where$Pe_{D_e} = Re_{D_e} P_r$, is the Peeklet number

Condition for laminar flow is

$$Pe_{D_e} <= \ Pe_{cr} \ \cong 2000$$

Liquid-metals have Prandtl number $P_r \ \cong 0.005 - 0.05$

Substituting $P_r = 0.05$ and $Pe_{cr} = 2000$ in equations (36) and (37), we get value of Nu_{D_e}as

$Nu_{D_e} = 7.9952679264$ for uniform (q/A), and

$Nu_{D_e} = 5.9952679264$ for uniform (t_0)

Calculation ofresults on substituting $P_r = 0.005$ and $Pe_{cr} = 2000$ in equations (36) and (37), are found as

$Nu_{D_e} = 7.157739336$ for (q/A) = uniform, and

$Nu_{D_e} = 5.157739336$for (r_0) = uniform

Conclusion:

(i) The results obtained by the present method are found in close agreement with that found earlier.

(ii) Value of Nu_{D_e} obtained for Slug flow under different conditions are found appreciably close to that of turbulent duct flow of liquid-metals when the heat transport by turbulent mixing is negligibly small.

(iii) Comparison of the value of Nu_{D_e} obtained for slug flow in circular tube with that calculated with the use of formulae (36) and (37) for liquid-metals performing turbulent flow in circular duct shows that the turbulent velocity can be well approximated by one of constant velocity.

(iv) Comparison of the results obtained shows that heat exchange between a liquid metal and a solid interface through turbulent boundary layer can be treated in the same way as for laminar boundary layer when the turbulent contribution to the heat exchange is negligibly small.

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